Existence and stability of breathers in a hexagonal lattice. Application in a dusty plasma crystal.

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I. Introduction

Recent theoretical and experimental investigations of dust-contaminated plasmas (*dusty plasmas*, DP) [1] have established the existence of *strongly coupled* DP lattices (crystals). These crystalline configurations, consisting of highly charged massive dust grains, are typically formed in the sheath region above a horizontal negatively biased electrode in gas discharge experiments (e.g. [1, 2]). Typical low-frequency oscillations are known to occur [1, 2] in these *mesoscopic* dust grain quasi-lattices in the longitudinal (in-plane, acoustic mode), horizontal transverse (in-plane) and vertical transverse (off-plane, inverse dispersive optic-like mode) directions. A variety of 2D and 3D configurations are possible [1b], although the spontaneous occurrence of successive *hexagonal* 2D layers seems to be the most often encountered possibility. A 1D DP crystal has also been realized experimentally, by using appropriate substrate potentials. Such 1D lattices have been shown to host collective excitations, in the form of solitons, localized envelope wavepackets, as well as discrete breather-type excitations (see [8] and Refs. therein).

In the present work a hexagonal DP lattice in considered. Transverse motion in this system is described by a Klein-Gordon-like Hamiltonian in the presence of an asymmetric quartic potential. By adopting real values for the potential (nonlinearity) parameters, as provided by experiments [3, 4, 5], and using the results of [6, 7], we shall prove that 2D DP crystals may support single-site as well as multi-site localised oscillations (multibreathers) [9].

II. Existence of Multibreathers in a Hexagonal Lattice

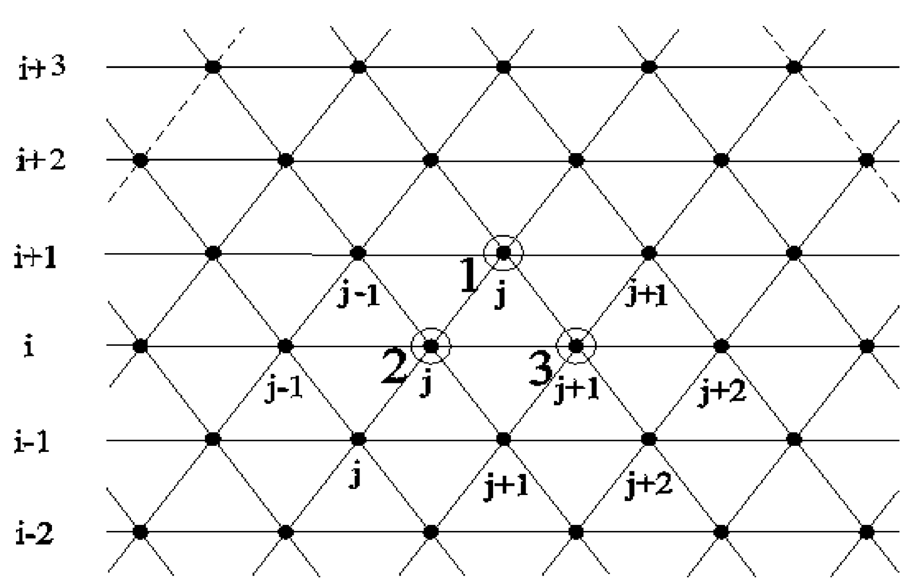

Fig.1 A hexagonal lattice

We consider the hexagonal lattice of fig.1, on site nonlinear potential *V(x)* and nearest neighbor linear interaction, with coupling parameter ε.

This system is described by a Klein-Gordon Hamiltonian of the form of eq.1

$$H = H_0 + \varepsilon H_1 = \sum_{i,j=-\infty}^{\infty} \frac{p_{ij}^2}{2} + V(x_{ij}) + \frac{\varepsilon}{2}\sum_{i,j=-\infty}^{\infty} [(x_{ij}-x_{i-1,j})^2 + (x_{ij}-x_{i-1,j+1})^2 + (x_{ij}-x_{i,j-1})^2 + (x_{ij}-x_{i,j+1})^2 + (x_{ij}-x_{i+1,j-1})^2 + (x_{ij}-x_{i+1,j})^2] \quad (1)$$

In [6] it is proven that, if the anharmonicity and the nonresonance with the linear spectrum conditions hold, this system supports multi-site breathers if

$$\frac{\partial\langle H_1\rangle}{\partial\phi_i} = 0 \quad (2) \qquad \text{and} \qquad \left|\frac{\partial\langle H_1\rangle}{\partial\phi_i\partial\phi_j}\right| \neq 0 \quad (3) \qquad \text{where} \qquad \langle H_1\rangle = \frac{1}{T}\oint H_1 dt$$

is the average value of H_1 along the unperturbed periodic orbit, T is the period of the resulting breather, and

$\phi_1 = w_2 - w_1,\ \phi_2 = w_3 - w_1 \Rightarrow \phi_3 = w_3 - w_1 = \phi_2 - \phi_1$ are the phase differences between the oscillators.

Since, the solution for a single oscillator can is an even function of the angle of the motion we can write

$x(t) = \sum_{n=0}^{\infty} A_n(J)\cos(nw) = \sum_{n=0}^{\infty} A_n(J)\cos[n(\omega t + \vartheta)]$. Then condition (2) becomes

$$\frac{\partial\langle H_1\rangle}{\partial\phi_1} = \frac{1}{2}\sum_{n=1}^{\infty} nA_n^2(\sin n\phi_1 - \sin n\phi_2) = 0, \qquad \frac{\partial\langle H_1\rangle}{\partial\phi_2} = \frac{1}{2}\sum_{n=1}^{\infty} nA_n^2(\sin n\phi_2 + \sin n\phi_3) = 0$$

which have always at least the solutions $\phi_1 = \phi_2 = 0,\ \phi_1 = 0\ \phi_2 = \pi,\ \phi_1 = \frac{2\pi}{3}\ \phi_2 = \frac{4\pi}{3}$

So, up to 3 moving oscillators we can distinguish the following 4 cases.

Case (a): *Single site breather*

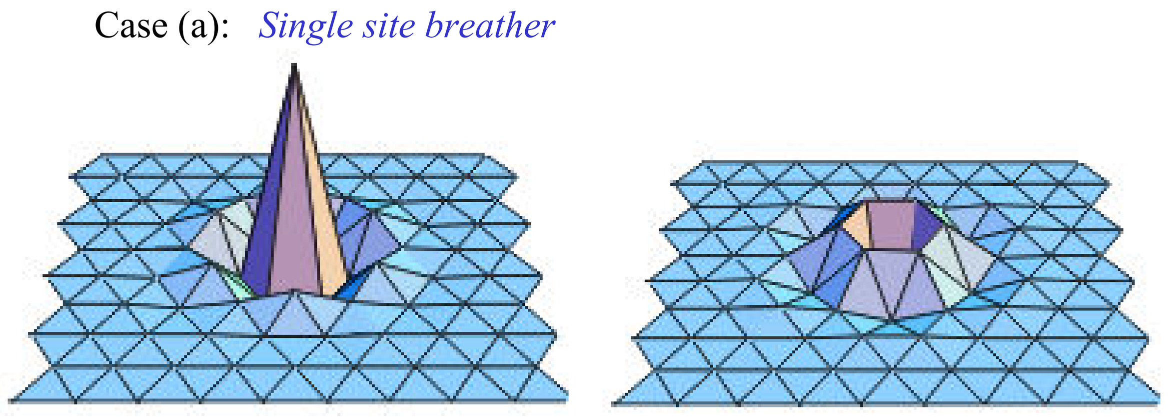
Fig.2: A single site breather.

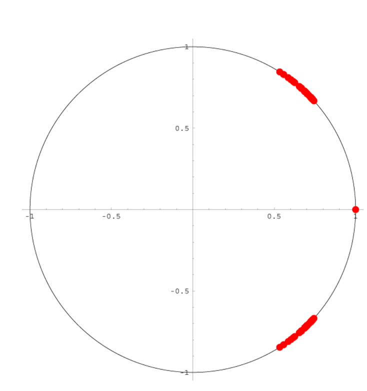
Fig3.: The Floquet mult. for the single breather for ε small

Case (b): $\phi_1 = \phi_2 = 0$ *In-phase multibreather*

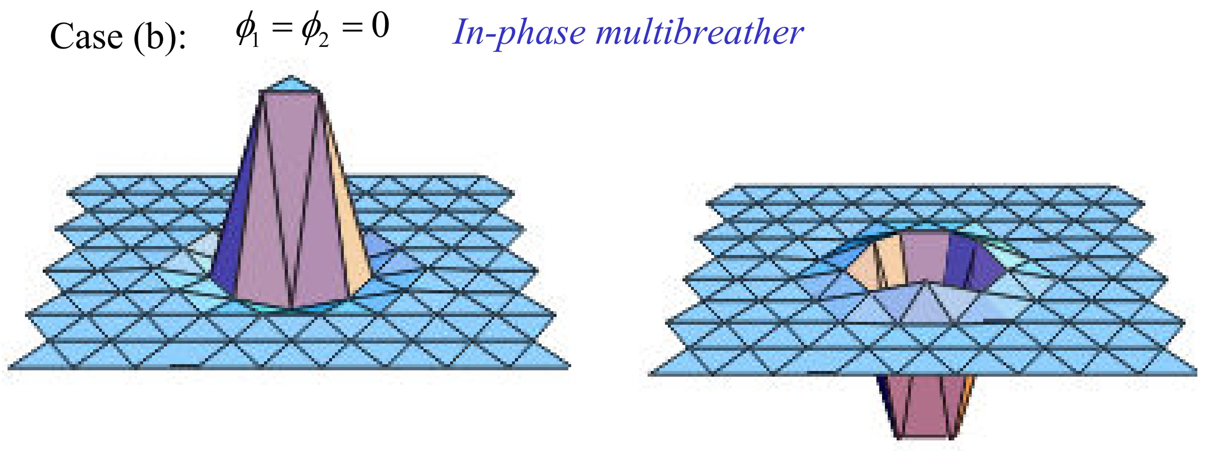
Fig.4: An in-phase 3-site breather

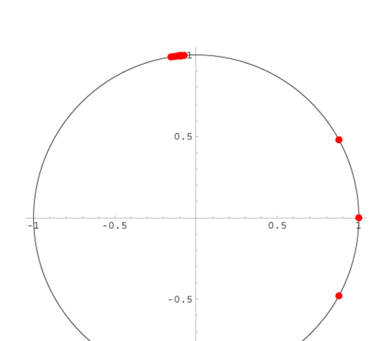
Fig5.: The Floquet mult. for the in-phase breather for [11] and ε small

In this case we have also the multipliers which correspond to the central oscillators. The condition for these multipliers to remain in the unit circle, which implies linear stability, at least for small ε is $\varepsilon\frac{\partial\omega}{\partial J} > 0$

Case (c): $\phi_1 = 0\ \phi_2 = \pi$ *Out of phase multibreather*

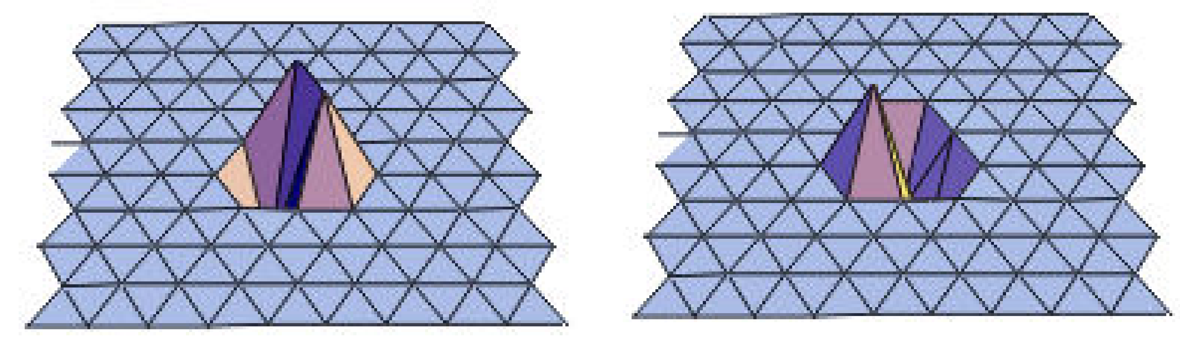
Fig.6: An out of phase 3-site breather

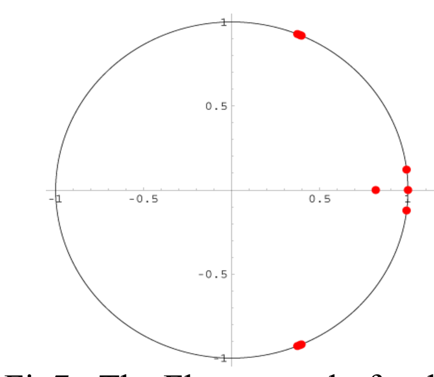
Fig7.: The Floquet mult. for the out of phase breather for [11] and ε small

The stability condition for ε small in this case is $\varepsilon\frac{\partial\omega}{\partial J} > 0$ and $\sum_{n=1}^{\infty}(-1)^n n^2 A_n^2 > 0$

Case (d): $\phi_1 = \frac{2\pi}{3}\ \phi_2 = \frac{4\pi}{3}$ *Vortex breather*

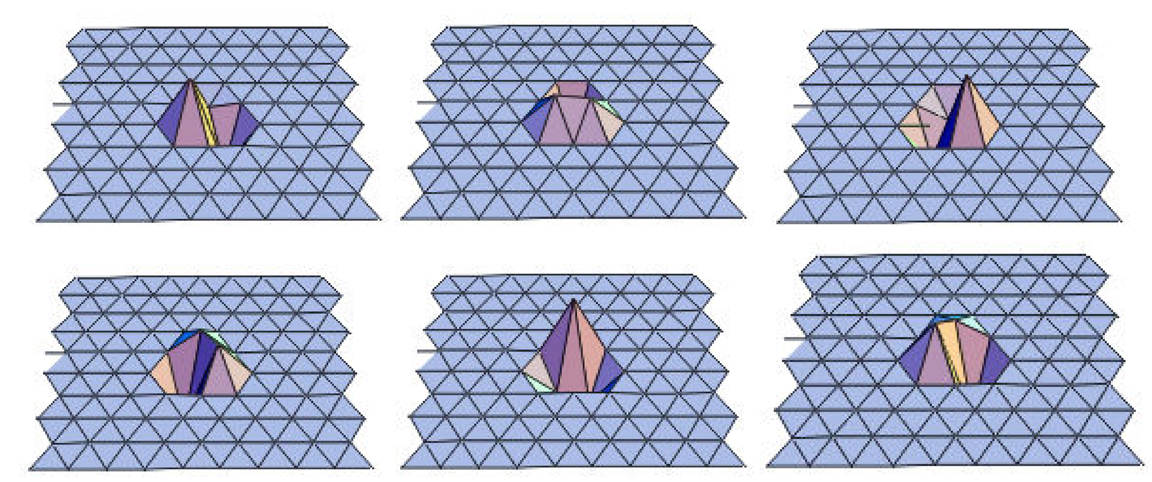
Fig.8: A vortex berather

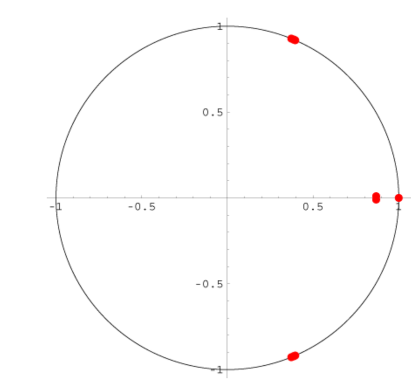
Fig9.: The Floquet mult. for the vortex breather for [11] and ε small

The stability condition for ε small in this case is $\varepsilon\frac{\partial\omega}{\partial J} f > 0$ with $f = \sum_{n=1}^{\infty}(-1)^n n^2 A_n^2 \cos\left(\frac{n\pi}{3}\right)$

III. The dusty plasma crystal

For negligible damping, and after the necessary normalization, the vertical, out of plane displacement in a DP crystal can be described by a Hamiltonian of the form (1) with ε<0 in account of the inverse dispersion of the lattice. The substrate potential can be approximate by a polynomial of the form

$$V(x) = \frac{x^2}{2} + a\frac{x^3}{3} + b\frac{x^4}{4}$$

In apersonal communication, Prof. Melzer suggested the set of values for *a,b,* ε which are shown in Table 1.

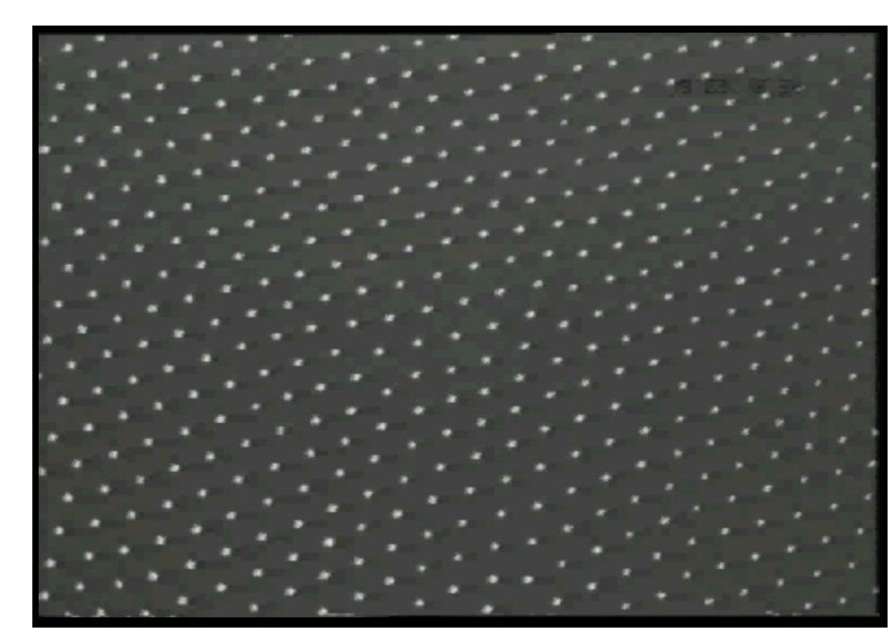
Fig.10: A camera snapshot of the crystalline phase of a DP

Table 1: Values suggested by A. Melzer

	a	b	c
set I	0.01	-0.04	0.034
set II	0.01	-0.06	0.065
set III	-0.21	-0.02	0.17

We consider only the first set of values since the other two suggest very large values of ε. Since $\varepsilon<0$ and $\frac{\partial\omega}{\partial J} < 0$ the out os phase and the vortex breathers immediately destabilize.

But the single and the in phase breathers can be continued for large enough values of ε in order for this system to support them.

IV. Conclusions – Future Work

We have shown that a two-dimensional hexagonal dusty plasma crystal can support single-site breathers, and in-phase 3-site breathers, while the rest of the theoretically predicted multi-site breathers are highly unstable. These results will hopefully be confirmed by appropriate experiments.

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[11]. Case I of Table 1 whith the parameter values suggested by A.Melzer (personal communication)

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